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DOI: <https://doi.org/10.24919/2522-4565.2022.49.33>

To cite this article: Shkarban, I. (2022). Lnhvistychnyi aspekt modalnosti suchasnoho anhlovnoho matematychnoho dyskursu [Linguistic aspect of modality of modern math discourse in English]. *Problemy humanitarnykh nauk: zbirnyk naukovykh prats Drohobytskoho derzhavnoho pedahohichnoho universytetu imeni Ivana Franka. Seriiia «Filolohiia» – Problems of Humanities. “Philology” Series: a collection of scientific articles of the Drohobych Ivan Franko State Pedagogical University*, 49, 231–236. doi: <https://doi.org/10.24919/2522-4565.2022.49.33>.

LINGUISTIC ASPECT OF MODALITY IN MODERN MATH DISCOURSE IN ENGLISH

Summary. The article reveals linguistic aspect of modality in modern math discourse in English, critically outlines a number of actual problematic issues in the area, such as the distinction between epistemic modality and evidentiality marked by formal logics philosophical grounding. General reference to previous scholarly activity in math modality research proves that it is largely based on propositional aspects of meaning. The math text corpus analysis aims to extract a set of modalities that are indispensable for formulating modal deductive reasoning. However, from a linguistic perspective academic math discourse requires natural language premise selection in the processes of mathematical reasoning and argumentation. It is presumed that two different self-attention cognition layers are focused at the same time on the proper classical symbolic logic and mathematical elements (formal language), while the other attends to natural language. Defining the semantic meanings of math discourse modality markers involves the interpretation phase. Thus, objectivity is generally associated with evidential adverbs which are markers of the evidence verification concerning the speaker's assessment of the truth value of the proposition. Modal auxiliaries of high, medium and low modality, semimodal verbs and conditionals involve ascribing a justification value in the set of possible logical inference making. The formal logical structure of mathematical reasoning explains the non-intuitive possibility of a deductive proof. It has been grounded that a linguistic category of modality in math discourse indispensably presupposes the universal truth of knowledge, high level of logical formalization in propositional verification status, formulaic nature of the argumentation, i.e. synthesis of hypothetical preconditions, theoretical knowledge and subjectivity of reasoning leading to a new hypothesis verification and visual exemplification of the empirical deductive processes in particular by linguistic means of modality expression.

Key words: epistemic modality; math discourse; linguistic aspect of modality; evidential adverbs; modal auxiliaries; propositional modality.

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Бібліографічний опис статті: Шкарбан, І. (2022). Лінгвістичний аспект модальності сучасного англійського математичного дискурсу. *Проблеми гуманітарних наук: збірник наукових праць Дрогобицького державного педагогічного університету імені Івана Франка. Серія «Філологія»*, 49, 231–236. doi: <https://doi.org/10.24919/2522-4565.2022.49.33>.

ЛІНГВІСТИЧНИЙ АСПЕКТ МОДАЛЬНОСТІ СУЧАСНОГО АНГЛОМОВНОГО МАТЕМАТИЧНОГО ДИСКУРСУ

Анотація. У статті розкрито лінгвістичний аспект модальності в сучасному англійському математичному дискурсі, критично окреслено ряд актуальних проблемних питань у

цій сфері, зокрема розрізнення епістемічної модальності та доказовості, зумовленої формальною логікою філософського обґрунтування. Загальний огляд досліджень доводить, що математична модальність значною мірою ґрунтується на класичній пропозиційній логіці суджень. Аналіз корпусу математичних текстів засвідчує вживання граматичних модальних категорій для формулювання модального дедуктивного міркування. З лінгвістичного погляду, академічний математичний дискурс вимагає дослідження передумов вживання природної мови в процесах математичного міркування та обґрунтування. Висунуто припущення щодо одночасної актуалізації двошарової когнітивної саморефлексії, зосередженої, з одного боку, на формальних логічних модальних операторах та математичних елементах (формальній мові), з другого, – на природній мові. Визначення семантичних значень маркерів модальності математичного дискурсу включає етап інтерпретації. Таким чином, доказові прислівники розглянуто в статті як маркери перевірки загальної об'єктивності суджень щодо оцінки мовцем істинності пропозиції. Допоміжні модальні мовні засоби високої, середньої та низької модальності, напівмодальні дієслова та вживання конструкцій умовного способу зумовлені семантикою ймовірності вибору серед гіпотетичної множини варіантів формально-логічного виведення. З'ясовано, що формально-логічна структура математичних міркувань пояснює неінтуїтивну можливість дедуктивного доведення. У статті обґрунтовано, що лінгвістична категорія модальності в математичному дискурсі обов'язково передбачає універсальну істинність знання, високий рівень логічної формалізації в статусі пропозиційної верифікації, формульний характер аргументації, тобто синтез гіпотетичних передумов, теоретичних знань та суб'єктивності міркування, що призводить до обґрунтування гіпотези та наочної ілюстрації емпіричних дедуктивних процесів, зокрема мовними засобами вираження модальності.

Ключові слова: епістемічна модальність, математичний дискурс, лінгвістичний аспект модальності, доказові прислівники, допоміжні модальні дієслова, пропозиційна модальність.

Formulation of the problem. Modal logic was originally defined in Aristotle's analysis as the logic of necessary and possible truths. In particular, a modality is any word or phrase that can be applied to a given statement to create a new statement that asserts the mode of truth of the preceding one: when, where, how it is true, or under which circumstances it may be true. Nowadays in the study of linguistics the mode of truth conditions of statements is deeply qualified in terms of knowledge, belief, temporal discourse, and ethics. Additionally, modality is defined as a grammatical category that allows the expression of aspects related to the speaker's attitude, that is degree of reliability, certainty, subjectivity, perspective, etc. (Dotsenko, 2006). Contrastive studies have revealed relevant differences in the use of modality across scientific discourse (Rozumko, 2017), but not all groups of modal markers have been studied systematically. This article focuses on linguistic aspects of math discourse modality, which has only recently begun to receive the scholarly attention (Goldblatt, 2003; Morante & Sporleder 2012; Lobovikov, 2021; Ferreira & Freitas, 2021). **The actuality of the paper** consists in discussing a number of issues which are central to the understanding of epistemic modality, such

as evidentiality, subjectivity, and objectivity. The subject matter of the research is linguistic aspect of math discourse modality markers in English.

The purpose of the article. This article seeks to address linguistic aspect of modality expression in modern math discourse in English, critically outline a number of problematic issues in the area, such as the distinction between epistemic modality and evidentiality marked by linguistic peculiarities of modality as grammatical category in English.

Research analysis. Propositional modality according to Palmer (1986) consists of two types: epistemic, "to express speaker's judgement about the factual status of the proposition", and evidential, "to indicate the evidence of the propositional factual status" (p. 8–13). There are also two types of event modality defined by Palmer (1986) as deontic, which relates to obligation or permission and to conditional factors considered external to the relevant by the individual, and dynamic, where the factors are supposed internal to the individual. Moreover, Palmer (1986) indicates other unreal categories found in the mood system: future, negative, interrogative, imperative-jussive, presupposed, conditional, purposive, resultative, wishes, and fears. The scholar also explains temporal and

aspectual relations of modality: all the categories concern the event reported by the utterance, whereas tense refers to the time of the event, aspect – to the nature of the event in terms of its “internal temporal constituency” (Palmer, 1986, p. 13–16).

Von Stechow (2006) defines modality from a philosophical standpoint as a category of linguistics meaning possibility and necessity expression, hence, a modalized sentence outlines an underlying prejacent proposition in the space of possibilities. Von Stechow (2006) provides analysis of several types of modal meaning, i.e. alethic, epistemic, deontic, bouletic, circumstantial, and teleological, and studies the cases proving that modal meaning is expressed by modal auxiliaries, semimodal verbs, adverbs, nouns, adjectives, and conditionals.

A more technical approach to modality is offered within the modal logic framework. Modal logic (Kripke, 1963; Garson, 2009) attempts to represent formally the reasoning involved in expressions of the type *it is necessary / possible that* starting from a weak logic. From modal logic standpoint, modality is analyzed in terms of “possible worlds semantics” (Kratzer, 1991, p. 640) as quantification over possible varieties. However, Kratzer (1991) argues that the meaning of modal expressions depends on the context, and therefore, is more complex than just quantifiers. Recent works on modality in the framework of a logically formalized axiomatic epistemology system in relation to mathematical knowledge prove that “mathematical knowledge as a whole is not an a priori system – rather, it is an empirical one” (Lobovikov, 2021, p. 223).

The main material. General reference to previous scholarly activity in math modality research is largely based on propositional aspects of meaning. One major theoretical issue that has dominated the field for many years concerns Kant’s apriorism underlying Hilbert’s formalism in the philosophical grounding of mathematics as a self-sufficing system, hence, proper mathematical propositions are always “judgements a priori, and not empirical, because they carry along with them the conception of necessity, which cannot be given by experience” (Lobovikov, 2021, p. 223). By scrutinizing the math text corpus, it is easy to extract a set of modalities that are indispensable for formulating modal deductive reasoning, that

is it is known that, it is a-priori known that, it is empirically known that, it is true that, it is provable that, it is consistent that, etc.

Goldblatt (2003) states that the first substantial algebraic analysis of modalized statements was carried out by Hugh MacColl, who symbolized the conjunction of two statements a and b by their concatenation ab , used $a+b$ for their disjunction, and wrote $a:b$ for the statement “ a implies b ”, which he said could be read “if a is true, then b must be true”, or “whenever a is true, b is also true”. The equation $a = b$ was given for the assertion that a and b are equivalent, meaning that each implies the other. However, while $a+b$ is a “necessary consequence” of $a : b$ (written $(a : b) : a + b$), he argued that the two formulas are not equivalent because their denials are not equivalent, claiming that the denial of $a:b$ “only asserts the possibility of the combination ab ”, while the denial of $a+b$ “asserts the certainty of the same combination” (p. 312).

It has also been suggested that modal symbolism as well as model theory have been generally applied to formalize reasoning about the programs behaviour in Computer Science to express “dynamical properties of transitions” between different states. While algebraic semantics interprets modal connectives as operators on the base of Boolean algebra, relational semantics uses relational structures, called Kripke models, which elements hypothetically outline possible worlds, moments of time, evidential situations, or states of a computer (Goldblatt, 2003, p. 310). For example, *in tense logic: eventually, henceforth, hitherto, previously, now, since, until, inevitably, ultimately, finally; in deontic logic: it is obligatory that, it is forbidden that, it is permitted that; in epistemic logic: it is known to that; it is generally accepted that; doxastic logic: it is believed that; in dynamic logic: after the program / computation / action starts / finishes, the program enables, throughout the computation; in geometric logic: it is locally the case that; metalogic: it is valid that, it is satisfiable that, it is provable that, it is consistent that, etc.* Thus, methods from both kinds of semantics have been used to research the nature of modal logic.

In recent years researchers (Morante & Sporleder, 2012; Rozumko, 2017; Ferreira & Freitas, 2021) dedicated several well-grounded

projects on modeling factuality, belief and certainty, detecting speculative sentences and hedging, identifying contradictions, and determining the scope of expressions of modality.

On the face of it, mathematics has no modal content, whereas mathematical statements are necessarily true. Nevertheless, from the empirical analysis of data in the text corpus by Werner (2016), it is apparent that modal auxiliaries of high, medium and low modality as well as epistemic adverbs are definitely applied in the modern math discourse in English. For example,

(1) *Obviously, we can rewrite the sets A and B as follows [...]* (Werner, 2016, p. 26)

(2) *It is immediately clear that equation (3.4) cannot have a solution since [...]* (Werner, 2016, p. 44)

(3) [...] *i.e. a root can be written as a power with a rational exponent [...]* (Werner, 2016, p. 56)

(4) *Thus, the logarithm of b to the base a is the power to which one must raise a to yield b.* (Werner, 2016, p. 59)

(5) *If a logarithm to another base should be computed, one can use the change-of-base formula for logarithms.* (Werner, 2016, p. 60)

(6) *For a root equation, we always have to test whether the solution(s) obtained indeed satisfy the given equation.* (Werner, 2016, p. 57)

Rozumko (2017) reports the fact that “markers of epistemic modality are concerned with the speaker’s assessment of the truth value of the proposition” (p. 75). At the same time, the character and degree of authorial representation are closely connected with the specific requirements of academic discourse of the discipline under consideration: in the natural sciences, the author tends to mark his presence minimally. Thus, on one hand, “modality represents the speaker’s angle” and markers of epistemic modality express “the speaker’s knowledge and commitment to the truth of the proposition” (Nuyts, 2001, p. 37), on the other hand, subjective and objective readings of epistemic adverbs are contextualized expressing objectivity with evidential adverbs, such as *obviously* and *It is immediately clear*, because they rely on verifiable evidence, that is “a commitment shared by other people (general knowledge)” (Nuyts, 2001, p. 64).

Hypothetically it can be claimed that examples (3), (5) confirm the epistemic possibility

and probability interpreted as objective, as the context suggests logical inference. Cases (4) and (6) allow speakers to “hint at some valid grounds” for mathematical operations, as well as “to convey an expectation that the stance would be seen as reasonable” (Wierzbicka, 2006, p. 259).

Conclusions and suggestions for further research. Within the scope of academic math discourse a set of significantly different modalities is called “knowledge” in the variety of contents which constitute multimodal epistemic logics. Given that, in the broadly accepted theories the modal logic treats “truth” as modality as well as provability. Consequently, many other modal terms, i.e. alethic, deontic, axiological, etc. are defined in epistemology concerning cognition, reasoning and grounding.

From a linguistic perspective academic math discourse requires natural language premise selection in the processes of mathematical reasoning and developing mathematical argumentation. It is presumed that two different self-attention layers must be focused on the proper classical symbolic logic and mathematical elements, such as expressions, variables, and artificial logic symbols, while the other attends to natural language features. Defining the semantic meanings of math discourse modality markers involves the interpretation phase. Thus, objectivity is generally associated with evidential adverbs, such as *indeed*, *obviously* and *clearly*, because they rely on verifiable evidence. In the studied cases evidential adverbs often introduce a statement which provides observations and supports a more general claim made in the previous one. Modal auxiliaries of high, medium and low modality, semimodal verbs and conditionals involve ascribing a justification value in the set of possible logical inference making. The formal logical structure of mathematical reasoning explains the non-intuitive possibility of a deductive proof. Thus, a linguistic category of modality in math discourse indispensably presupposes the universal truth of knowledge, high level of logical formalization in propositional verification status, formulaic nature of the argumentation, i.e. synthesis of hypothetical preconditions, theoretical knowledge and subjectivity of reasoning leading to a new hypothesis grounding or visual exemplification of the empirical deductive

processes, two self-attention layering as a sign of stylistic correlation of mathematical signs, symbols, and logical operators within the natural language communication sphere.

Further research may focus on establishing the ratio of modal auxiliaries, semimodal verbs and

epistemic adverbs in the modern math discourse in particular for teaching English for academic purposes. It would be interesting to investigate their frequencies in math analysis to examine the ratio of personal judgement versus the rigoristic formulaic impersonality of math discourse.

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